On the “non-perturbative analysis” of zero-temperature dephasing: I. Dyson equation and self energy.

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We point out that the structure of the self-energy suggested in cond-mat/0208140 as a result of a “non-perturbative analysis” by “purely mathematical means” is incompatible with the very definition of the self-energy.

Recent paper [1] by Golubev and Zaikin, called a “reply” to our comment [2], contains nothing new but attempts to justify and improve the calculation of the preexponential factors for some “path integrals” having nothing to do with the contributions identified in the second part of Ref. [2]. For this reason, we do not accept it as a response to our objections, and will not analyze the calculation of Ref. [1] here. However, since GZ misrepresent basic notions of field theory methods [3] rather than only their applications to disordered systems, we find it necessary to make several comments. Below, we will show that Eqs. (5) and (8) of Ref. [1] [copied below as Eqs. (2) and (3)], used by GZ to explain “why perturbative in the interaction techniques are insufficient for the problem in question”, contradict the construction of Dyson equation. For pedagogical reasons we recall ABC of Dyson self-energy first, and then analyse GZ arguments.

ABC of Dyson self-energy — According to Dyson [3], any propagator, such as an electron Green function $G(\omega)$, or a Cooperon in the present case, is connected to its bare form, $G_0(\omega)$, and a self energy $\Sigma(\omega)$

$$G(\omega) = \frac{1}{G_0^{-1}(\omega) - \Sigma(\omega)}, \tag{1}$$

where ω is a schematic notation for energies and momenta of the particle or Cooperon. It should be emphasized that Σ is not an “ambiguous” quantity defined by an expression $\Sigma = 1/G_0 - 1/G$ but rather is a well defined mathematical object – sum of all *one-particle irreducible* graphs, see Fig. 1 (a-c) for the lowest order contributions.

The significance of the introduction of Σ is the following: the expansion of Eq. (1) in powers of the interaction strength contains poles of higher and higher orders, $G_0^n(\omega)$.

In contrast, $\Sigma(\omega)$ includes one particle irreducible graphs only and does *not* contain contributions proportional to $G_0^n(\omega)$ – each of the contributions is a non-factorizable integral over internal momenta and energies. These integrations either eliminate or substantially reduce the singularities of Σ. In particular, when the integrals are ultraviolet (determined by large energies and momenta and proportional to some power of the high-energy cut-off) $\Sigma(\omega)$ is finite at $\omega \to 0$, and its expansion in powers of the interaction coupling constant is a well defined asymptotic series.

The conventional scheme does not assume that one determines $G(\omega)$ by some means (non-perturbative analysis) and then evaluates $\Sigma(\omega)$ through Eq. (1). Quite contrary, if $G(\omega)$ can be expanded in terms of the interaction strength, each of these terms should be also possible to obtain from the diagrammatic expansion for $\Sigma(\omega)$. As long as the two approaches give different results, it is the ”non-perturbative analysis” rather than the diagrammatic expansion to be questioned.

Golubev-Zaikin's self-energy – Let us discuss the self energy of the Cooperon, presented by GZ in Ref. [1]: (Eq. (5) of Ref. [1])

$$\Sigma(\omega) = \frac{(\alpha + \beta T)^2 - i\omega\beta T}{2\alpha + \beta T - i\omega}, \tag{2}$$

“where α and β are proportional to interaction strength[1]”. Since $\Sigma(\omega)$ from Eq. (2) at any finite ω can be expanded in terms of α and β, *it is perturbative and should be accessible by usual diagrammatic approach.* On the other hand this expansion is singular at $\omega \to 0$: (Eq. (8) of Ref. [1])

$$\Sigma(\omega) = \beta T + \frac{\alpha^2}{-i\omega} + \dots, \tag{3}$$

note that each term in this expansion is proportional to a power of interaction constant. Once again, Eqs. (2) and (3) are the equations used by GZ to demonstrate that the “ perturbative in the interaction techniques are insufficient for the problem in question”.

In GZ scheme, α is determined by ultraviolet integral and proportional to the high energy scale $1/\tau_e$, which is supposed to be much bigger than $T, 1/\alpha$ and any other

scale in the problem [4]. The form of the second term in the right hand side of Eq. (3) implies that this term
(i) is the second order expansion in α of the self-energy, see Fig. 1(b-c);
(ii) contains the same pole as bare Green function (bare Cooperon);
(iii) is determined by factorizable product of the ultraviolet divergent integrals.

As we have already explained such answer is hardly feasible for a one particle irreducible self energy. In the second order of the perturbation theory, there is an object which has a chance to possess such a structure, see Fig. 1(d). However, it does not belong to the *self-energy.*

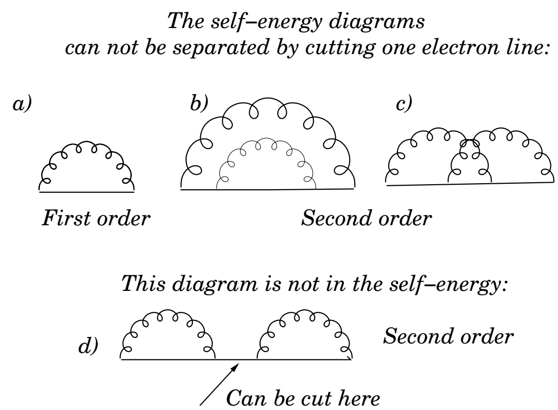

FIG. 1: First (a) and second order (b-d) diagrams.

Conclusion — The equation for the self energy presented by GZ *(i)* has a well defined perturbative expansion in the interaction and *(ii)* contradicts the conventional diagrammatic methods. Therefore, the arguments of GZ about insufficiency of the perturbative expansion lack the substance.

Such a discussion could be only justified, provided that GZ explicitly evaluate the irreducible diagrams of topology of Fig. 1(b-c), and demonstrate that $1/\omega$ divergence appears in a double ultraviolet integral. We do not require a full scale second order calculation of all the diagrams with numerical coefficients, we would like to see at least one **irreducible** graph which is double ultraviolet and $1/\omega$ divergent at the same time.

[1] D.S. Golubev and A.D. Zaikin, cond-mat/0208140.
[2] I.L. Aleiner, B.L. Altshuler and M.G. Vavilov, cond-mat/0110555.
[3] F.Dyson, Phys. Rev. **75**, 1736 (1949); canonical textbook is A.A. Abrikosov, L.P. Gorkov, and I.E. Dzyaloshinskii, *Methods of Quantum field theory In Statistical Physics*, (Prentice-Hall, Englewood Cliffs, NJ, 1963).
[4] GZ still did not reveal their method of evaluating the cutoff of their non-logarithmically divergent ultraviolet integrals except by declaring that the "divergence of the integral at high frequencies is cut at $\Omega \sim 1/\tau_e$ because a classical path needs a time exceeding τ_e to return to the same point" [D.S. Golubev, A.D. Zaikin, G. Schön, cond-mat/0110495, p.8] (here Ω is the integration frequency within self-energy).